



Oxford Cambridge and RSA

Wednesday 15 May 2019 – Morning

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **70**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} at^2$$

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Show that the equation $x = 7 + 2x^2$ has no real roots.

[3]

2 In this question you must show detailed reasoning.

Fig. 2 shows the graphs of $y = 4 \sin x^\circ$ and $y = 3 \cos x^\circ$ for $0 \leq x \leq 360$.

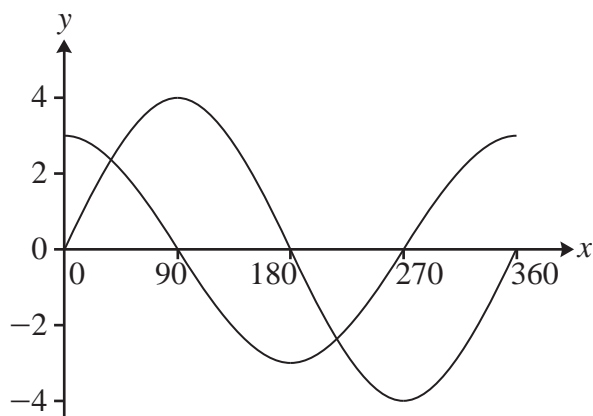


Fig. 2

Find the x -coordinates of the two points of intersection, giving your answers correct to 1 decimal place.

[3]

3 Given that k is an integer, express $\frac{3\sqrt{2}-k}{\sqrt{8}+1}$ in the form $a+b\sqrt{2}$ where a and b are rational expressions in terms of k .

[4]

4 A triangle ABC has sides $AB = 5$ cm, $AC = 9$ cm and $BC = 10$ cm.

(a) Find the cosine of angle BAC, giving your answer as a fraction in its lowest terms.

[2]

(b) Find the exact area of the triangle.

[3]

- 5 In this question, the unit vectors \mathbf{i} and \mathbf{j} are horizontal and vertically upwards respectively.

A particle has mass 2.5 kg.

- (a) Write the weight of the particle as a vector. [1]

The particle moves under the action of its weight and two external forces $(3\mathbf{i} - 2\mathbf{j})$ N and $(-\mathbf{i} + 18\mathbf{j})$ N.

- (b) Find the acceleration of the particle, giving your answer in vector form. [2]

- 6 Fig. 6 shows a train consisting of an engine of mass 80 tonnes pulling two trucks each of mass 25 tonnes.

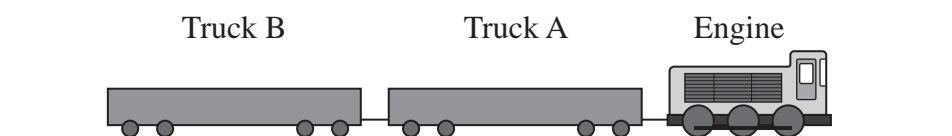


Fig. 6

The engine exerts a driving force of D N and experiences a resistance to motion of 2000 N. Each truck experiences a resistance of 600 N. The train travels in a straight line on a level track with an acceleration of 0.1 m s^{-2} .

- (a) Complete the force diagram in the Printed Answer Booklet to show all the forces acting on the engine and each of the trucks. [3]
- (b) Calculate the value of D . [2]
- (c) The tension in the coupling between the engine and truck A is larger than that in the coupling between the trucks. Determine how much larger. [2]

- 7 In this question you must show detailed reasoning.

- (a) Nigel is asked to determine whether $(x+7)$ is a factor of $x^3 - 37x + 84$. He substitutes $x = 7$ and calculates $7^3 - 37 \times 7 + 84$. This comes to 168, so Nigel concludes that $(x+7)$ is not a factor.

Nigel's conclusion is wrong.

- Explain why Nigel's argument is not valid.
- Show that $(x+7)$ is a factor of $x^3 - 37x + 84$. [2]

- (b) Sketch the graph of $y = x^3 - 37x + 84$, indicating the coordinates of the points at which the curve crosses the coordinate axes. [5]

- (c) The graph in part (b) is translated by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Find the equation of the translated graph, giving your answer in the form $y = x^3 + ax^2 + bx + c$ where a , b and c are integers. [4]

8 In this question you must show detailed reasoning.

Show that the only stationary point on the graph of $y = x^2 - 4\sqrt{x}$ is a minimum point at $(1, -3)$. [7]

9 In this question you must show detailed reasoning.

A car accelerates from rest along a straight level road. The velocity of the car after 8 s is 25.6 m s^{-1} .

In one model for the motion, the velocity $v \text{ m s}^{-1}$ at time t seconds is given by $v = 1.2t^2 - kt^3$, where k is a constant and $0 \leq t \leq 8$.

(a) The model gives the correct velocity of 25.6 m s^{-1} at time 8 s. Show that $k = 0.1$. [2]

A second model for the motion uses constant acceleration.

(b) Find the value of the acceleration which gives the correct velocity of 25.6 m s^{-1} at time 8 s. [2]

(c) Show that these two models give the same value for the displacement in the first 8 s. [5]

10 In this question you must show detailed reasoning.

(a) Sketch the gradient function for the curve $y = 24x - 3x^2 - x^3$. [5]

(b) Determine the set of values of x for which $24x - 3x^2 - x^3$ is decreasing. [2]

11 David puts a block of ice into a cool-box. He wishes to model the mass m kg of the remaining block of ice at time t hours later. He finds that when $t = 5$, $m = 2.1$, and when $t = 50$, $m = 0.21$.

(a) David at first guesses that the mass may be inversely proportional to time. Show that this model fits his measurements. [3]

(b) Explain why this model

(i) is not suitable for small values of t , [1]

(ii) cannot be used to find the time for the block to melt completely. [1]

David instead proposes a linear model $m = at + b$, where a and b are constants.

(c) Find the values of the constants for which the model fits the mass of the block when $t = 5$ and $t = 50$. [3]

(d) Interpret these values of a and b . [2]

(e) Find the time according to this model for the block of ice to melt completely. [1]

END OF QUESTION PAPER

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